APPLICATION OF FREQUENCY METHODS FOR STUDYING NONSTATIONARY REGIMES IN THE FUNCTIONING OF HEAT-ENGINEERING SYSTEMS

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A method is proposed for numerical calculation of complex systems of partial and ordinary differential equations, based on the use of the integral Laplace and Fourier transform.

The efficiency with which certain industrial installations are used is determined by stability requirements and operational characteristics within very strict limits independent of the conditions of use [1, 2]. In this case, the most difficult aspect to realize is the maintenance of a strict temperature regime (within the limits of tenths of a degree) with constantly changing variables during the time that heat loads are applied. In order to fulfill this requirement, it is necessary to create complicated heat-engineering systems that consist of a large number of various kinds of elements, which, in its turn, has led to the development of mathematical models and techniques for their realization, allowing for a quite precise calculation of the parameters of these systems not only in stationary but also in dynamic regimes of operation.

Mathematical models in this case consist of complicated systems of nonlinear and linear, partial and ordinary differential equations, describing the functional dependence of the variables on two and more parameters. Solving such systems with the help of traditional analytic methods, which allow in principal for solving the problem or reducing it to one solved earlier, is practically impossible. Realization of similar models with the help of numerical methods gives good results in calculating a specific version of a given scheme of a heat-engineering system, but encounter serious difficulties in carrying out the analysis and synthesis due to the limited size of the memory and operational speed of modern computers.

For this reason, in solving the problems of analysis and especially synthesis, it is most useful to use a method based on the use of integral Laplace and Fourier transforms, combining in an optimal manner the possibilities of analytical and numerical studies of complex systems. We will illustrate the essence of the method with an example of a heat-engineering system (Fig. 1), the mathematical model for which consists of the following system of equations:

heat-exchange equation for the object being controlled

$$\frac{\partial T_1(x,\ \tau)}{\partial \tau} + V_1 \frac{\partial T_1(x,\ \tau)}{\partial x} = k_1 [T_{12}(x,\ y,\ \tau) - T_1(x,\ \tau)]\,; \tag{1}$$

$$\frac{\partial T_{13}(x,\ y,\ \tau)}{\partial \tau} = a_1 \frac{\partial^2 T_{13}(x,\ y,\ \tau)}{\partial y^2} + k_2 [T_{12}(\tau) - T_{13}(x,\ y,\ \tau)] + k_3 [q_1(\tau) - \varepsilon \sigma_0 T_{13}^4(x,\ y,\ \tau)]\,; \tag{2}$$

heat-transfer equation in the main ducts

$$\frac{\partial T_i(x,\ \tau)}{\partial \tau} + V_{\mathrm{d},i} \frac{\partial T_i(x,\ \tau)}{\partial x} = k_{\mathrm{d},i} [T_{\mathrm{W},i}(x,\ \tau) - T_i(x,\ \tau)]\,; \tag{3}$$

$$\frac{dT_{\mathrm{W},i}(x,\ \tau)}{d\tau} = k_{\mathrm{W},i} [T_i(x,\ \tau) - T_{\mathrm{W},i}(x,\ \tau)]\,, \tag{4}$$

where $i = 1, 2, \ldots, n$ is the number of ducts;

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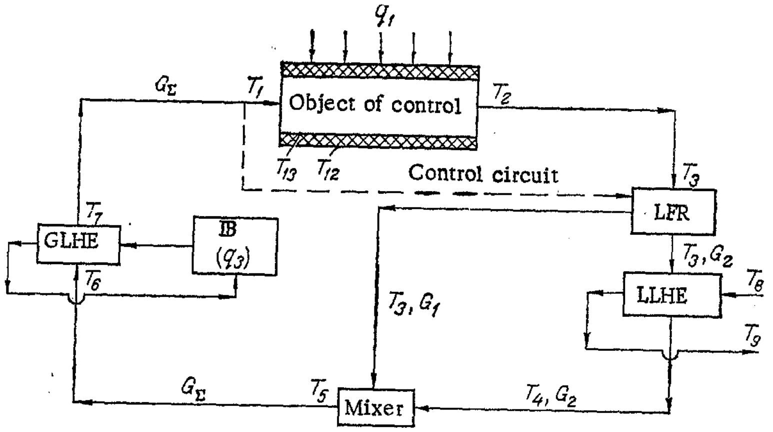

Fig. 1. Functional diagram of a heat-engineering system: IB) instrument block; LFR) liquid flow regulator; GLHE, LLHE) gas–liquid and liquid–liquid heat exchangers.

equations for the elements of the regulator circuits

$$\frac{dU(\tau)}{d\tau} = \frac{k_{se}}{T_{se}} T_1(0,\ \tau) - \frac{1}{T_{se}} U(\tau)\ ; \tag{5}$$

$$\frac{d\bar{\varphi}(\tau)}{d\tau} = \frac{k_{dr}}{T_{dr}} U(\tau)\ ; \tag{6}$$

$$G_2(\tau) = k_{lfr}\bar{\varphi}(\tau)\ ; \tag{7}$$

heat-exchange equation in the liquid—liguid heat exchanger

$$\frac{dT_4(\tau)}{d\tau} = k_4[T_3(\tau) - T_4(\tau)] - k_5 q_2(\tau)\ ; \tag{8}$$

$$\frac{dT_9(\tau)}{d\tau} = k_6 q_2(\tau) - k_7[T_9(\tau) - T_8(\tau)]\ ; \tag{9}$$

$$\frac{dq_2(\tau)}{d\tau} = k_8\frac{dT_3(\tau)}{d\tau} - k_9\frac{dT_8(\tau)}{d\tau} - k_{10}\frac{dG_2(\tau)}{d\tau}\ ; \tag{10}$$

equation for the mixer

$$T_5(\tau) = T_3(\tau) - \frac{G_2(\tau)}{G_\Sigma}[T_3(\tau) - T_4(\tau)]; \tag{11}$$

heat-exchange equation in the instrument block

$$\frac{dT_{10}(\tau)}{d\tau} = k_{11}[T_{11}(\tau) - T_{10}(\tau)] + k_{12} q_3(\tau)\ ; \tag{12}$$

heat-exchange equation in the gas—liquid heat exchanger

$$\frac{dT_{11}(\tau)}{d\tau} = k_{13}[T_{10}(\tau) - T_{11}(\tau)] - k_{14} q_4(\tau)\ , \tag{13}$$

$$\frac{dT_7(\tau)}{d\tau} = k_{15} q_4(\tau) - k_{16}[T_7(\tau) - T_6(\tau)]; \tag{14}$$

$$\frac{dq_4(\tau)}{d\tau} = k_{17}\frac{dT_{10}(\tau)}{d\tau} - k_{18}\frac{dT_6(\tau)}{d\tau} \tag{15}$$

with the corresponding boundary conditions, characteristic for closed control systems.

In accordance with the method being proposed, the starting system of equations (1)-(15) is linearized relative to the stationary regime with respect to a small parameter and written in the Laplace representation.

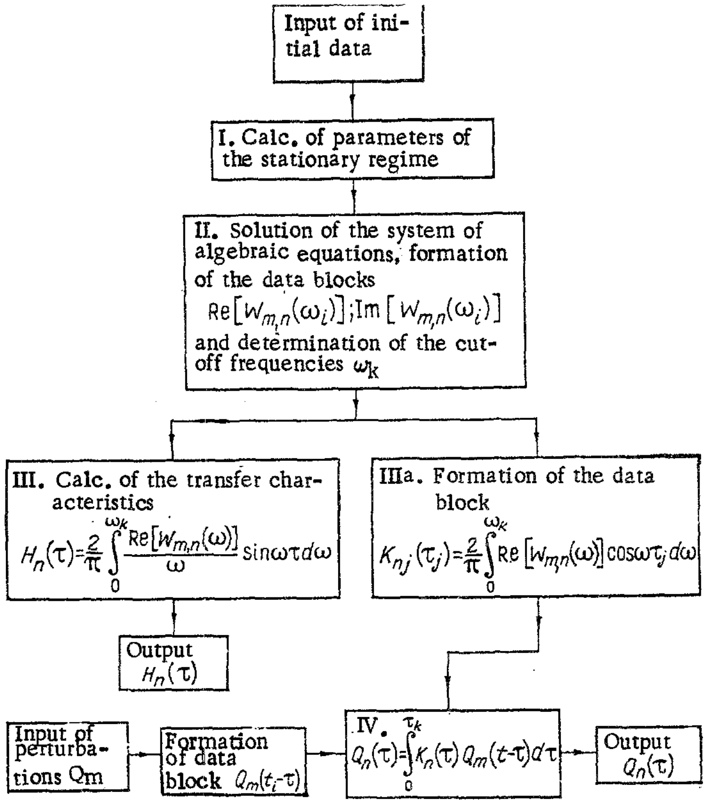

Fig. 2. Block diagram of the program for calculating the parameters of a heat-engineering system.

In the representation for elements with distributed parameters, we seek a solution relative to the spatial coordinates, and then the system is represented in the complex plane by replacing the operator p by iω. As a result of the separation of the real and imaginary parts of the equations describing the reaction of the heat-engineering system being considered to a specific perturbation, in the complex plane, we obtain algebraic equations of the following form:

$$\operatorname{Re}[Q_n(\omega)] = \sum_{m=1}^{k} \{\operatorname{Re}[W_{mn}(\omega)] \operatorname{Re}[Q_m(\omega)] - \operatorname{Im}[W_{mn}(\omega)] \operatorname{Im}[Q_m(\omega)]\}; \quad (16)$$

$$\operatorname{Im}[Q_n(\omega)] = \sum_{m=1}^{k} \{\operatorname{Im}[W_{mn}(\omega)] \operatorname{Re}[Q_m(\omega)] + \operatorname{Re}[W_{mn}(\omega)] \operatorname{Im}[Q_m(\omega)]\}, \quad (17)$$

where Q_n and Q_m are the parameter being determined and the determining parameter; W_{mn} is the transfer function corresponding to the perturbation.

These equations are easily solved with the help of a computer with a minimum amount of machine time in comparison with the expenditures of machine time necessary for solving the starting system of equations (1)-(15).

The problem of determining the values of the parameters necessary to carry out the analysis and synthesis of the heat-engineering system being considered with the help of the algebraic equations (16)-(17) in accordance with the block diagram shown in Fig. 2 is solved in two stages. In the first stage, we determine the values of the real and imaginary parts of the transfer function of the open system, which are then used to study the stability of the system, the values of the cut-off frequencies, the real and imaginary parts of the transfer function of the closed system, and also its amplitude and phase frequency characteristics. At the second stage, the values of the transfer process control parameter are determined according to the values of the real or imaginary parts of the transfer function of the closed system by calculating the corresponding integral relation.

Therefore, as a result of solving Eqs. (16)-(17), with minimum expenditures of machine time it is possible to obtain the values of all the parameters necessary for carrying out the analysis and synthesis of the system being investigated.

Thus, from the example examined above, it follows that the proposed method significantly simplifies the problem of realizing the complex mathematical models and can be effectively used for studying quite complex heat-engineering systems. In addition, the proposed method can be successfully used for developing methods and systems for automated system design that provide heat regimes for various purposes.

NOTATION

T, temperature; V, rate of motion of the heat-exchange agent; $k_1 = \alpha_1 \Pi_1 / c_1 \rho_1 g S_1$; α_1, heat-transfer coefficient; Π_1, perimeter of the heat-exchange agent channel; c_1, specific heat capacity of the heat-exchange agent; ρ_1, density; g, acceleration due to gravity; S_1, area of the transverse cross section of the heat-exchange agent channel; x and y, spatial coordinates; τ, time; a_1, thermal diffusivity; $k_2 = \nu / c_2 \rho_2 g r$; ν, relative insulation area; c_2, specific heat capacity of the wall; ρ_2, density of the wall; r, thermal resistance of the insulation; $k_3 = (1-\nu)/c_2 \rho_2 g$; q, specific heat flux; ε, emissivity; σ_0, Stefan–Boltzmann constant; $k_{d,i} = (\alpha_1 \Pi_1)_{d,i} / (c_1 \rho_1 g S_1)_{d,i}$; $k_{w,i} = (\alpha_1 \Pi_1)_{d,i} / (c_2 \rho_2 g F)_{w,i}$; $F_{w,i}$, area of the transverse cross section of the duct wall; k_{se}, coefficient of amplification of the sensing element; T_{se}, time constant for the sensing element; U, voltage; k_{dr}, drive amplification coefficient; T_{dr}, drive time constant; k_{lfr}, coefficient for amplification of the liquid flow regulator; G, weight flow rate of the heat-transfer agent; $\overline{\varphi}$, relative deviation of the regulator; $k_8 = (dq_2/dT_3)_0$; $k_9 = (dq_2/dT_8)_0$; $k_{10} = (dq_2/dG_2)_0$, coefficient of sensitivity for liquid–liquid heat exchanger according to the input parameters; $k_4 = k_7 = k_{11} = k_{13} = k_{16} = 1/\tau_{0,j}$; $\tau_{0,j}$, time spent by the heat-transfer agent in the j-th element; $k_5 = k_6 = k_{12} = k_{14} = k_{15} = 1/c_1 \cdot g m_j$; m_j, mass of the heat-transfer agent in the j-th element; $k_{17} = (dq_4/dT_{10})_0$; $k_{18} = (dq_4/dT_6)_0$, coefficient of sensitivity of the gas–liquid heat-exchange agent according to the input parameters) W, transfer function. The indices are as follow: d, ducts; w, walls, se, sensing elements; dr, drive; lfr, liquid flow regulator.

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